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Simplified model of transient radiative cooling of spherical body

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ABSTRACT

The cooling of a spherical body in case when radiation is the only heat transfer mechanism between the body surface and the environment has been considered. A mathematical process model employing an own approximate kinetic equation of heat conduction has been worked out. Thus, the process has been described by an ordinary differential equation combined with an algebraic one. The proposed simplified model has been compared with the exact solution and with approximate relations presented by Su [4]. It has been found that the proposed model is accurate and employable for both long and short times as well as for any relations between outer and inner heat transfer resistances.

1. Introduction

In some cases, e.g. in outer space technologies as well as in cryogenic engineering, the surface transmitting the heat is surrounded with vacuum (heat loss from space vehicles, heat loss from vacuum insulations). Therefore, the mechanism of heat convection does not exist and radiation is the only heat transfer mechanism between the body surface and the environment. In this paper the cooling of a spherical body in the described case has been considered.

This problem was analysed previously [1–4]. Transient cooling problem with conduction and radiation for a hot spherical body covered with a cold coating material was considered in [1]. Modest [2] discussed the performance of a simple longitudinal rectangular-fin radiator used to reject heat from a tubular space craft. A different problem exists in case of occurrence of convection as an additional to radiation heat transfer mechanism during heating/cooling. A case such as that for heating of a spherical body (droplet) was analysed by Sazhin [5,6].

If radiation is the only heat transfer mechanism between the body surface and the environment the distributed process model is based on the equation of transient heat conduction in sphere with boundary condition for surface in the form of Stefan–Boltzmann equation for radiation. The basic difficulty in solving equations of this model is the fact of existence of partial differential equation with non-linear boundary condition. The numerical solving of these equations is time-consuming particularly in case of recurrent calculations. For this reason it is convenient to simplify the mathematical process description by elimination of spatial coordinate inside the body. In such case an ordinary differential equation can be employed for mathematical formulation of the problem.

Campo and Villaseñor [3] compared the results of solution of distributed model of the process with the results of solution of so called classical lumped model based on the assumption of uniform temperature of spherical body. The sink temperature in this case was taken as zero. The temporal variations of relative errors of both the average body temperature as well as the boundary to center temperature ratio were compared. The terms for which the employment of classical lumped model gives the approvable errors were fixed.

Su [4] improved the classical lumped model by introducing a distinction between the average body temperature and the boundary temperature. Two models presented by Su are better compatible with the distributed model than the classical lumped model.

In this paper for description of radiative cooling of a spherical body an own simplified relationship, based on polynomial approximation, has been employed. The employment of this relationship leads, like in the cases of approximations employed by Su, to elimination of spatial coordinate from process model i.e. to formulation of problem with an ordinary differential equation. The proposed way of approximate calculations has been numerically tested by comparison with the exact solution.

The main advantage of presented approximations is the rate of calculations: the length of time of calculations in this case is

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Nomenclature

$\overline{A}$, A	dimensionless temperatures defined by Eqs. (3) and (4)
$Bi(= hR/k)$	Biot number
c_p	specific heat of body, J kg^{-1} K^{-1}
h	heat transfer coefficient, W m^{-2} K^{-1}
k	thermal conductivity of body, W m^{-1} K^{-1}
N_{rc}	radiation-conduction parameter (Eq. (21))
N_{rc}^*	parameter (Eq. (15))
r	spatial coordinate, m
R	radius of spherical body, m
T	temperature, K
t	time, s

Greek symbols

$\alpha(= k/(c_p\rho))$	thermal diffusivity, m^2 s^{-1}
δ	relative error (Eq. (48))
Δ	relative error (Eq. (47))
ε	surface emissivity
η	dimensionless spatial coordinate (Eq. (6))
σ	Stefan-Boltzmann constant ($= 5.67\cdot10^{-8}$ W m^{-2} K^{-4})
θ	dimensionless temperature (Eq. (19))
τ	dimensionless time (Eq. (10))
ρ	density, kg m^{-3}

Subscripts and superscripts

ex	exact
i	initial
s	sink
0	body center
1	body surface
$\bar{}$	average value

considerably less than in the case for the distributed model. Also a good accuracy of calculations is observed.

Some practical examples of application of the considered case of radiative cooling in vacuum are: thermal sprays for rapid solidification and determination of thermophysical properties of spherical bodies levitating in outer space.

Though a case of cooling of a spherical body has been considered in this paper there is a possibility of generalization of the simplified relationship for other body shapes (slab, cylinder).

2. General relationships

If a temperature profile in spherical body is known, an average body temperature can be determined on the base of the following balance equation (T_{ref} – any reference temperature):

$$\frac{4}{3}\pi R^3\rho c_p\left(\overline{T}-T_{\text{ref}}\right) = \int_0^R 4\pi r^2\rho c_p\left(T-T_{\text{ref}}\right)dr \tag{1}$$

which results in the following equation:

$$\overline{T} = \frac{3}{R^3}\int_0^R r^2 T dr \tag{2}$$

A dimensionless variable $\overline{A}$, characterizing a ratio of the heat amount emitted out by body from process beginning to the total heat amount possible for emission, has been defined:

$$\overline{A} = \frac{T_i-\overline{T}}{T_i-T_s} \tag{3}$$

Analogically, a local dimensionless variable A characterizing a ratio of the local heat amount emitted out to the total heat amount possible for emission has been defined:

$$A = \frac{T_i-T}{T_i-T_s} \tag{4}$$

The following relation between A and $\overline{A}$ can be easily obtained:

$$\overline{A} = 3\int_0^1 \eta^2 A d\eta \tag{5}$$

where η is a dimensionless spatial coordinate:

$$\eta = \frac{r}{R} \tag{6}$$

3. Distributed model

The heat conduction in sphere is described by the following equation:

$$\frac{\partial T}{\partial t} = \frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) \tag{7}$$

The initial condition has the form:

$$T = T_i \text{ for } t = 0 \tag{8}$$

From sphere surface the heat is transported to environment by radiation. Hence the following boundary condition for surface arises [7]:

$$-k\left(\frac{\partial T}{\partial r}\right)_{r=R} = \varepsilon\sigma\left(T_1^4-T_s^4\right) \tag{9}$$

When introducing the dimensionless spatial coordinate η and the dimensionless time τ:

$$\tau = \frac{\alpha t}{R^2} \tag{10}$$

the equation of heat conduction can be converted into the form:

$$\frac{\partial A}{\partial \tau} = \frac{1}{\eta^2}\frac{\partial}{\partial \eta}\left(\eta^2\frac{\partial A}{\partial \eta}\right) \tag{11}$$

with initial condition:

$$A = 0 \text{ for } \tau = 0 \tag{12}$$

and boundary condition:

$$\left(\frac{\partial A}{\partial \eta}\right)_{\eta=1} = N_{rc}^*\left[(A_i-A_1)^4-(A_i-1)^4\right] \tag{13}$$

where the constants A_i and N_{rc}^* are defined as follows:

$$A_i = \frac{T_i}{T_i-T_s} \tag{14}$$

$$N_{rc}^* = \frac{\varepsilon\sigma R(T_i-T_s)^3}{k} \tag{15}$$

The parameter N_{rc}^* characterizes a ratio of the heat conduction resistance to the heat radiation resistance. When the heat conduction resistance equals zero ($k \to \infty$), then $N_{rc}^* = 0$. In such case the rate of body cooling is controlled only by heat radiation resistance. Contrary, when the heat radiation resistance equals zero, then $N^*_{rc} \to \infty$, and the rate of body cooling is controlled only by heat conduction resistance. The parameter N^*_{rc} plays an analogical role as the Biot number in case of convectional heat transport between the body surface and the environment. If $N^*_{rc} \to \infty$ (similarly if $Bi \to \infty$) the condition $A_1 = 1$ is valid instead of boundary condition (13) and the analytical solution of Eq. (11) has the known form [8]:

$$\overline{A} = 1 - \frac{6}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-n^2\pi^2\tau\right) \tag{16}$$

When differentiating Eq. (5) towards time and taking into account the equation of heat conduction (11) one can get:

$$\frac{d\overline{A}}{d\tau} = 3\left(\frac{\partial A}{\partial \eta}\right)_{\eta=1} \tag{17}$$

The value of derivative on body surface is a result of boundary condition (13). Hence:

$$\frac{d\overline{A}}{d\tau} = 3N^*_{rc}\left[(A_i - A_1)^4 - (A_i - 1)^4\right] \tag{18}$$

In items [3] and [4] a case of zero sink temperature ($T_s = 0$) is considered and the dimensionless temperature is defined as follows:

$$\theta = \frac{T}{T_i} \tag{19}$$

For $T_s = 0$, the relations between $\overline{A}$ and $\overline{\theta}$ as well as between A and θ are following: $\overline{\theta} = 1 - \overline{A}$ and $\theta = 1 - A$. For $T_s = 0$ Eq. (18) becomes simplified to the form:

$$\frac{d\overline{\theta}}{d\tau} = -3N_{rc}\theta_1^4 \tag{20}$$

where:

$$N_{rc} = \frac{\varepsilon\sigma R T_i^3}{k} = A_i^3 N^*_{rc} \tag{21}$$

4. Classical lumped model and Su improved lumped models

In classical lumped model the body temperature is assumed to be uniform at any time, i.e. $\theta = \overline{\theta}$ (also $A = \overline{A}$). In such case, after separation of variables and after integration of Eq. (20), one can get (for $A_i = 1$):

$$\overline{\theta} = 1 - \overline{A} = (1 + 9N_{rc}\tau)^{-1/3} \tag{22}$$

Su [4] introduced to classical lumped model an improvement consisting in distinction between the average body temperature and the boundary temperature. For description of temperature profile in sphere he employed the Hermite one-side and two-side approximations. The one-side approximation of $H_{2,0}/H_{0,0}$ type led to the equation of I Su model:

$$N_{rc}\theta_1^4 + 8\theta_1 - 8\overline{\theta} = 0 \tag{23}$$

The two-side approximation of $H_{2,1}/H_{0,0}$ type led to the equation of II Su model:

$$N_{rc}\theta_1^4 + 5\theta_1 - 5\overline{\theta} = 0 \tag{24}$$

The above algebraic equations, resulting from the employment of Hermite approximations, were utilised for numerical solving the differential Equation (20). In estimation presented in [4] both approximations give more exact results in comparison with classical lumped model where $\theta_1 = \overline{\theta}$ but the employment of the two-side approximation (Eq. (24)) results in smaller errors.

It has been proved in Appendix A that the assumption of parabolic temperature profile in the sphere [9–11]: $T = a_0 + a_2r^2$ where a_0 and a_2 are temporal functions, leads to the relationship (A.6) which combined with the boundary condition (13) gives the relation:

$$N^*_{rc}\left[(A_i - A_1)^4 - (A_i - 1)^4\right] = 5(A_1 - \overline{A}) \tag{25}$$

For $A_i = 1$ this equation is identical with relation (24) proposed in [4]. Taking into consideration in Eq. (17) the relation (A.6) one can get:

$$\frac{d\overline{A}}{d\tau} = 15(A_1 - \overline{A}) \tag{26}$$

For $A_1 = 1$ the above relation is known as the Glueckauf equation or the Linear Driving Force (LDF) equation (12) and is often employed in problems of mass transfer kinetics (adsorption).

If convection is the only heat transfer mechanism between the body and the environment the relation (27) is valid instead of the boundary condition (13):

$$\left(\frac{\partial A}{\partial \eta}\right)_{\eta=1} = Bi(1 - A_1) \tag{27}$$

When combining this relation with (A.6) one can get a formula describing the sphere surface dimensionless temperature:

$$A_1 = \frac{\overline{A} + 0.2Bi}{1 + 0.2Bi} \tag{28}$$

Then taking into account this relation in Eq. (26) one can obtain after integration:

$$\overline{A} = 1 - \exp\left(-\frac{15\tau}{1 + 5/Bi}\right) \tag{29}$$

Dombrovsky and Sazhin [11] utilised the parabolic temperature profile to describe the heating of spherical droplet. They analysed the runs of the dimensionless surface temperature A_1 in the function of the product $Bi\cdot\tau$ for various Bi values. They found the courses corresponding with the analytical solution except the range of small τ values. A similar conclusion can be seen as a result of the work on the analysis of analogical diffusion process in porous adsorbent pellet [9]: $\tau > 0.05$ was accepted as the validity criterion for the above relations.

5. Approximate kinetic equation of heat conduction

Approximate kinetic equations are often employed for modeling kinetics of transfer of component in adsorption processes [9,10,12–14]. In consideration of analogy between the heat and the mass transfer processes these equations can be successfully employed for modeling heat conduction processes.

The temporal derivative of $\overline{A}$ is given by differentiation of Eq. (16):

$$\frac{d\overline{A}}{d\tau} = 6\sum_{i=1}^{\infty}\exp\left(-n^2\pi^2\tau\right) \tag{30}$$

For long times ($\overline{A}\rightarrow 1$)the series in formulas (16) and (30) are quick-convergent and the first summand is important. Thus:

$$\frac{d\overline{A}}{d\tau} = 6\exp(-\pi^2\tau) = \pi^2(1-\overline{A}) \tag{31}$$

For short times ($\overline{A}\rightarrow 0$) the formula (16) becomes simplified as follows [8]:

$$\overline{A} = 6\sqrt{\frac{\tau}{\pi}} - 3\tau \tag{32}$$

When differentiating this relation one can get:

$$\frac{d\overline{A}}{d\tau} = \frac{3}{\sqrt{\pi\tau}} - 3 = \frac{3}{1-\sqrt{1-\frac{\pi}{3}\cdot\overline{A}}} - 3 \tag{33}$$

Using the approximation $\sqrt{1-x} \cong 1-\frac{1}{2}x$, valid for small x values, it is possible to write:

$$\frac{d\overline{A}}{d\tau} \cong \frac{3}{1-\left(1-\frac{\pi}{6}\cdot\overline{A}\right)} - 3 = \frac{18}{\pi\overline{A}} - 3 \tag{34}$$

In this paper a simplified kinetic equation has been employed. This equation is based on the following function:

$$K = \left[\frac{d\overline{A}/d\tau}{1-\overline{A}} - \pi^2\right]\cdot\overline{A} = \left(k_h - \pi^2\right)\cdot\overline{A} \tag{35}$$

The expression $k_h = (d\overline{A}/d\tau)/(1-\overline{A})$ acts as the heat transfer coefficient on the analogy of mass transfer coefficient in [14]. In accordance to (31) for $\overline{A}\rightarrow 1$the implication $k_h = \pi^2$ occurs as well as

$$\lim_{A\rightarrow 1} K = 0 \tag{36}$$

It results from (34) and (35) that for small values of $\overline{A}$:

$$K = \frac{\frac{18}{\pi} - 3A}{1-A} - \pi^2 A \tag{37}$$

So:

$$\lim_{A\rightarrow 0} K = \frac{18}{\pi} \tag{38}$$

The variation of the function $K = f_{ex}(\overline{A})$ is depicted in Fig. 1 in the form of signs. This variation corresponds with analytical solution of equation of heat conduction in sphere with boundary condition of first type ($A_1 = 1$). The approximate equation has been based on the substitution of the function $K = f_{ex}(\overline{A})$ with third degree polynomial:

$$K \cong c_0 + c_1(1-\overline{A}) + c_2(1-\overline{A})^2 + c_3(1-\overline{A})^3 \tag{39}$$

The relations (36) and (39) give $c_0 = 0$ while relations (38) and (39) lead to the result:

$$c_1 + c_2 + c_3 = \frac{18}{\pi} \tag{40}$$

Eq. (39) can be written in the form:

$$K = \left(\frac{18}{\pi} - c_2 - c_3\right)(1-\overline{A}) + c_2(1-\overline{A})^2 + c_3(1-\overline{A})^3$$

After transformations one can obtain:

$$F = -(c_2 + 2c_3) + c_3\overline{A} \tag{41}$$

where

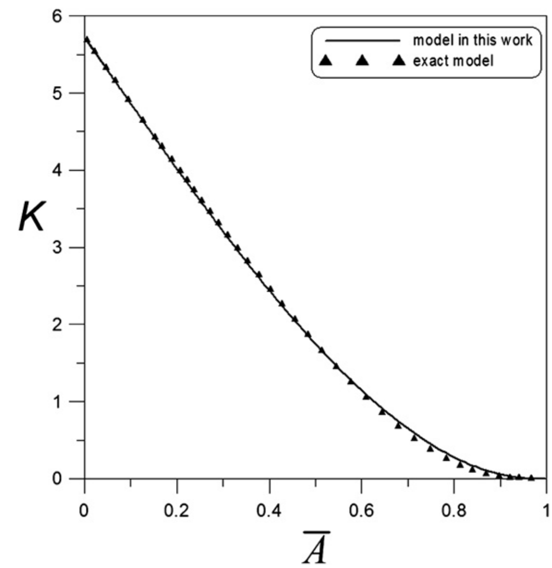

Fig. 1. Comparison of exact (distributed model) and approximate (proposed model) relations between K and $\overline{A}$.

$$F = \frac{1}{\overline{A}}\left(\frac{K}{1-\overline{A}} - \frac{18}{\pi}\right) \tag{42}$$

The variation of F vs. $\overline{A}$, resulting from the exact solution of conduction equation, is depicted in Fig. 2. This variation has been approximated using a linear function (relation (41)). Parameters of the line have been determined: $c_2 + 2c_3 = 2.8850$, $c_3 = -3.1705$. Then the following values have been obtained: $c_0 = 0$, $c_1 = -0.3259$, $c_2 = 9.2260$, $c_3 = -3.1705$. The variation of K vs. $\overline{A}$, resulting from the polynomial approximation, is depicted in Fig. 1 in the form of solid line. As can be seen, the run of approximating function coincides with the exact one.

The approximate kinetic equation has been obtained by comparison of right sides of Eqs. (35) and (39). It has the form:

$$\frac{d\overline{A}}{d\tau} = \left[\pi^2 + \frac{1}{\overline{A}}\sum_{j=1}^{3} c_j(1-\overline{A})^j\right]\cdot(1-\overline{A}) \tag{43}$$

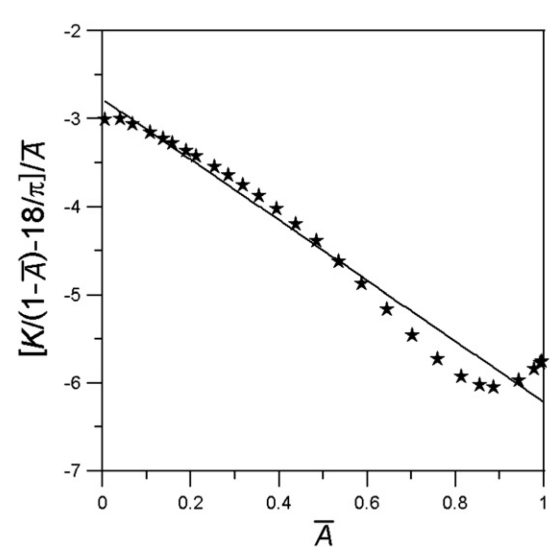

Fig. 2. Determinations of constants in simplified kinetic equation.

The main advantage of employment of Eq. (43) is the fact of occurrence of small errors tending to zero for both the long and the short process times. Contrary, the employment of LDF equation for short times encumbers the results with considerable errors which results from:

$$\lim_{\tau\to 0}\frac{\overline{A}_{\mathrm{LDF}}-\overline{A}_{ex}}{\overline{A}_{ex}} = \lim_{\tau\to 0}\left[\frac{1-\exp(-15\tau)}{\frac{6}{\sqrt{\pi}}\sqrt{\tau}-3\tau}-1\right] = -1 \tag{44}$$

where the integrated LDF equation (26) (for $A_1 = 1$) and the relation (32) have been utilised.

The relation (43) generalised for $A_1 < 1$ has the form:

$$\frac{d\overline{A}}{d\tau} = \left[\pi^2+\frac{1}{A_1\overline{A}}\sum_{j=1}^{3}c_jA_1^{2-j}(A_1-\overline{A})^j\right]\cdot(A_1-\overline{A}) \tag{45}$$

When combining relation (45) with boundary condition (13) one can obtain:

$$\left[\pi^2+\frac{1}{A_1\overline{A}}\sum_{j=1}^{3}c_jA_1^{2-j}(A_1-\overline{A})^j\right]\cdot(A_1-\overline{A}) = 3N_{rc}^*\left[(A_i-A_1)^4-(A_1-1)^4\right] \tag{46}$$

From the algebraic Eq. (46) one should calculate A_1. Details of solution are presented in Appendix B. If A_1 is determined one can integrate numerically the ordinary differential Eq. (45).

6. Algorithms of calculations

Because all calculations were applied to $A_i = 1$, then $N_{rc}^* = N_{rc}$. Data for calculations were various values of N_{rc}. Results of calculations are relationships between $\overline{A}$ and τ. In numerical calculations in the first place the results obtained on the base of proposed model with the results of exact calculations have been compared. For comparisons also the runs of relationships between $\overline{A}$ and τ both for Su models and for classical lumped model have been determined. The following models have been considered:

1. Distributed (exact) model

For exact solution of Eq. (11) with initial condition (12) and boundary condition (13) a procedure based on Crank-Nicolson scheme [15] has been employed. To obtain $\overline{A}$ value it was necessary to apply the integration in accordance with relation (5). The Simpson method has been employed. The values of function have been calculated using the Lagrange interpolative formula.

2. Simplified model proposed in this work

Eq. (45) has been integrated numerically using the Runge-Kutta method. Values of A_1 have been calculated numerically from relation (B.1) using the Newton method.

3. Su models

Eq. (18) has been integrated numerically using the Runge-Kutta method. Values of A_1 have been calculated according to algebraic Eq. (25), solved numerically using the Newton method (II Su model). Because the I and the II Su models differ only in a multiplier, in I Su model the constant equal to 5 in formula (25) one should substitute with the constant equal to 8.

4. Classical lumped model

The relation (22) has been utilised in the calculations.

7. Results of calculations

The presented simplified model of radiative cooling of a spherical body has been verified numerically. As a measure of deviations between the numerical values obtained by simplified model (*app*) and the values obtained by distributed model (*ex*) two types of relative errors, denoted as Δ and δ, have been employed. The relative error Δ was defined as follows:

$$\Delta = \frac{\overline{A}_{app}-\overline{A}_{ex}}{\overline{A}_{ex}} \tag{47}$$

where $\overline{A}_{ex}$ is a value determined from the distributed model.

Definition of error δ is different:

$$\delta = \frac{(1-\overline{A})_{app}-(1-\overline{A})_{ex}}{(1-\overline{A})_{ex}} \tag{48}$$

The employment of relative errors defined by Eq. (48) makes possible a comparison between deviations of presented simplified model and deviations of Su models [4] because for $T_s = 0$ the $(1-\overline{A})$ values correspond with dimensionless temperatures $\overline{T}/T_i$.

7.1. Comparison of accuracy of considered models

Fig. 3 shows temporal variations of $\overline{A}$ values predicted by both the distributed model and the presented simplified model for various values of radiation-conduction parameter N_{rc}. The higher value of radiation-conduction parameter, the greater is the value of $\overline{A}$ for a given time. For the extreme case when $N_{rc}\to\propto$ the exact run has been determined on the base of Eq. (16). Moreover, it can be observed that the approximate and exact values are very close in the whole range of N_{rc} values. Therefore, the proposed simplified model is not limited in employment only to good heat conductors. The model predicts accurate results in extreme cases: for very small and very large $(N_{rc}\to\propto)$ values of radiation-conduction parameter.

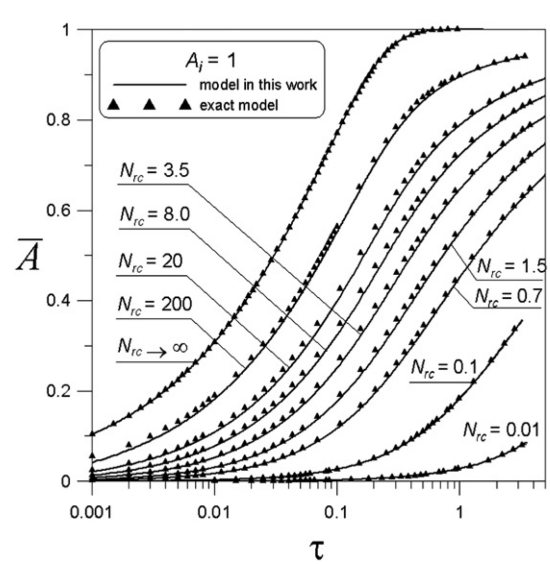

Fig. 3. Temporal variation of $\overline{A}$ for various N_{rc}.

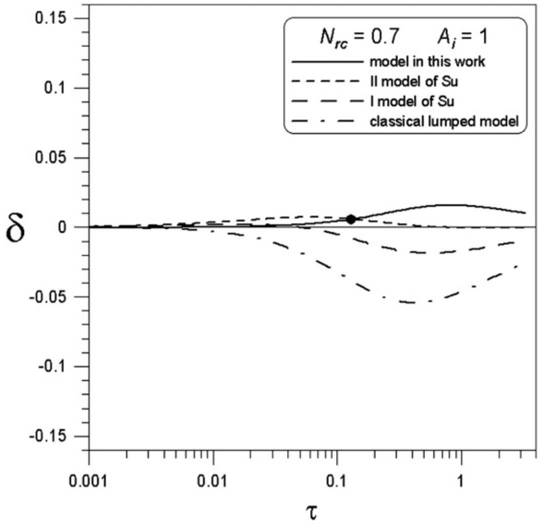

Fig. 4. Temporal variation of δ at $N_{rc} = 0.7$ for various models.

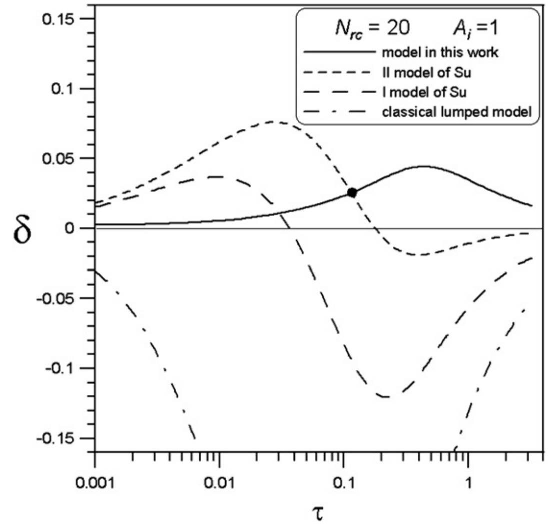

Fig. 6. Temporal variation of δ at $N_{rc} = 20$ for various models.

Figs. 4–6 present relations between the relative error δ, defined by Eq. (48), and the dimensionless time τ for various values of radiation-conduction parameter N_{rc}.

Fig. 4 displays deviations between the values predicted by various compared models for $N_{rc} = 0.7$. The lowest deviations, less than 1%, gives II Su model. The simplified model proposed in this work gives a maximum deviation of 1.6%. I Su model gives a maximum deviation of 1.8%. The classical lumped model gives a maximum deviation above 5%. It can be observed that for each analysed model a maximum deviation from the exact value occurs for different value of dimensionless time. Moreover, the simplified model presented in this work gives only positive deviations whereas the classical lumped model gives only negative ones. The Su models give both positive and negative deviations.

Fig. 5 displays results for $N_{rc} = 3.5$. For this value of parameter a maximum error of the simplified model presented in this work is equal to 3.2% and is the smallest from among all errors of analysed approximate models. The classical lumped model cannot be employed in this case because of its maximum deviation equal about 16%.

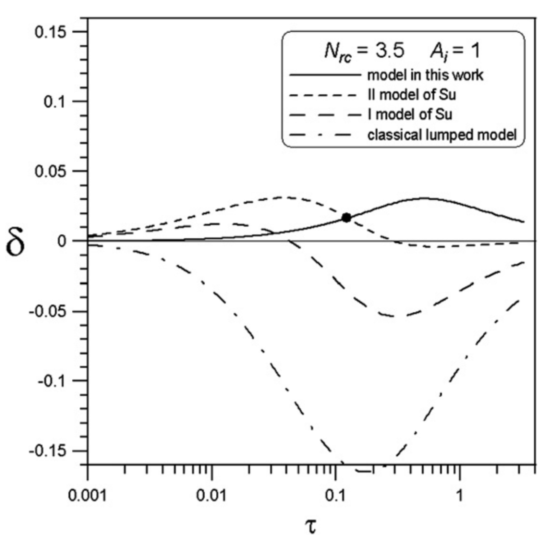

Fig. 5. Temporal variation of δ at $N_{rc} = 3.5$ for various models.

The model presented in this work gives acceptable errors δ even for $N_{rc} = 20$ (4.4%). For this order of magnitude of this parameter not any model is competitive – it can be seen in Fig. 6. However, its accuracy decreases with an increase of radiation-conduction parameter i.e. the deviations increase. But the increase is slight. For this reason the proposed model is peerless for large N_{rc} values – thus it can be employed for any N_{rc} value.

Table 1 gives the maximum deviations defined by Eq. (48) for particular models and for various values of parameter N_{rc}.

Figs. 4–6 show that for II Su model the maximum errors occur within the range of $0.01 < \tau < 0.1$ whereas for the proposed models within the range of $0.1 < \tau < 1$. Therefore, it would be advisable to employ in calculations both mentioned models: for short times – the model presented in this work, for longer times – the II Su model. Table 2 presents the values of maximum errors referred to the described hybrid method of calculations and the limiting time for which the maximum errors occur. The average value of limiting time equals about 0.12. In Figs. 4–6 one can observe the limiting points.

Fig. 7 illustrates a temporal variation of relative errors Δ defined by Eq. (47). When comparing this variation with the variation presented in Fig. 5 (referring to the same value of N_{rc}) one can observe the fundamental differences. For short times both Su models give considerable relative errors Δ. The shorter time, the errors are larger. Contrary to this, the values of Δ for results predicted by the proposed model are small and tend towards zero at $\tau \to 0$. Different values of errors Δ and δ appear as a consequence of different definitions: quantity $\overline{A}$ (the base for calculation of Δ) determines the amount of heat emitted out by body while $(1-\overline{A})$ (the base for calculation of δ) determines the heat amount possible for emission which is dependent on the average body temperature. For short times the quantities $\overline{A}$ are small. It results in large relative

Table 1
Maximum values of δ for various models at $A_i = 1$.

N_{rc}	δ_{max}, %			
	Classical lumped model	I Su model	II Su model	Proposed model
0.7	−5.4	−1.8	0.7	1.6
1.5	−9.6	−3.2	1.6	2.3
3.5	−16.5	−5.3	3.1	3.0
8.0	−25.0	−8.0	5.2	3.7
20	−35.7	−12.0	7.7	4.4
200	–		14	4.6

Table 2
Maximum values of δ for hybrid method ($A_i = 1$).

N_{rc}	δ_{max}, %	τ_{limit}
0.7	0.6	0.130
1.5	1.1	0.128
3.5	1.6	0.125
8.0	2.1	0.120
20	2.5	0.115
200	2.9	0.105

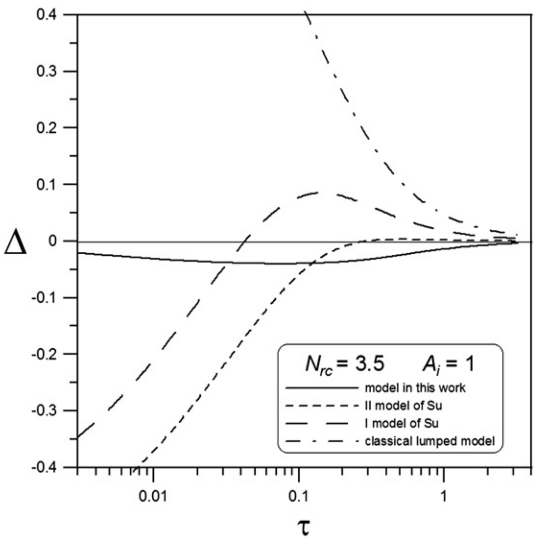

Fig. 7. Temporal variation of Δ at $N_{rc} = 3.5$ for various models.

errors Δ. Contrary to this, the quantities $(1-\overline{A})$ are then large (close to one). Therefore, the relative errors δ are small. Large relative errors Δ of Su models for small values of τ appear as a consequence of relation (44) referred to error value for $\tau \rightarrow 0$.

7.2. Surface temperature

A precise determination of body surface temperature is of fundamental importance in modeling of process of radiative body cooling because the amount of heat emitted is a function of fourth power of the temperature. Fig. 8 shows a temporal variation of $A_1 = (T_i - T_1)/(T_i - T_s)$ determined on the base of both the proposed model and other models analysed in this paper. The comparison refers for $N_{rc} = 3.5$. It can be observed that the classical lumped model predicts quite different values from those of distributed model. A similar discrepancy can be observed for A_1 values determined on the base of Su models for times less than $\tau = 0.1$. Contrary, the model proposed in this work predicts values close to those of predicted by distributed model in the whole range of time.

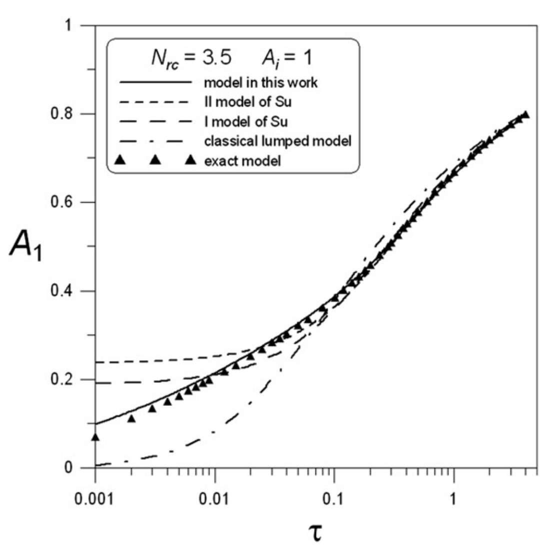

Fig. 8. Comparison of temporal variations of A_1 for various models.

8. Conclusions

1. The simplified kinetic Eq. (43) describes the heat conduction in spherical body at a constant boundary temperature and predicts the results close to the exact ones. This equation also can be adapted for the case of existence of heat transfer resistance both inside and outside the body.
2. The employment in model of radiative cooling of spherical body of simplified Eq. (43) worked out by the authors gives the results consistent with the exact solution in the whole range of radiation-conduction parameter. For short process times the error caused by the simplification tends towards zero.
3. Taking as a comparative criterion the relative error δ defined by Eq. (48) it has been proved that the simplified model proposed in this work predicts more accurate results than the Su models, particularly for short cooling times ($\tau < 0.1$) and for large values of the radiation-conduction parameter ($N_{rc} > 3.5$). The employment of this model is not limited only to good heat conductors.
4. Taking as a comparative criterion the relative error Δ defined by Eq. (47) the presented model predicts results with good accuracy. For $\tau = 0$ and $\tau \rightarrow \infty$ the relative errors Δ tend towards zero. It is advantageous that the proposed model gives small errors Δ for short times ($\tau < 0.1$) because the Su models within this range give considerable errors Δ and make any interpretation of quantity $\overline{A}$ value impossible.
5. The solution of equation of presented model is much more simple than the solving of equation of distributed model, i.e. a differential equation with non-linear boundary condition. In the proposed model one should solve an ordinary differential equation combined with an algebraic one.
6. The II Su model [4] described by relation (24) for $A_i = 1$ is consistent with the assumption of parabolic temperature profile inside the body. Therefore, it is consistent with the approximate kinetic equation which is known as LDF equation.
7. The classical lumped model assuming a uniform body temperature at any time (flat temperature profile) can be employed only for limited range of N_{rc} values (practically for $N_{rc} < 0.7$). Outside this range the model gives very large errors.
8. For $N_{rc} < 3.5$ and $\tau > 0.12$ the proposed model gives worse accuracy than the II Su model. Then for $N_{rc} < 3.5$ it is advisable to employ a combine (hybrid) method of calculations consisting in employing of the proposed model for $\tau < 0.12$ and the II Su model for $\tau > 0.12$.

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Appendix A: Model based on approximation of temperature profile with parabolic equation

If the temperature in sphere center is denoted as T_0 and on sphere boundary as T_1 one can get:

$$T = T_0 + (T_1 - T_0)\left(\frac{r}{R}\right)^2 \tag{A.1}$$

When introducing the dimensionless variables A and η one can obtain:

$$A = A_0 + (A_1 - A_0)\eta^2 \tag{A.2}$$

Eq. (5) leads to the result:

$$\overline{A} = 3\int_0^1 \left[A_0 + (A_1 - A_0)\eta^2\right]\eta^2 d\eta = \frac{2}{5}A_0 + \frac{3}{5}A_1 \tag{A.3}$$

Therefore:

$$A_0 = \frac{5}{2}\overline{A} - \frac{3}{2}A_1 \tag{A.4}$$

When differentiating the profile equation (A.2) one can get:

$$\frac{\partial A}{\partial \eta} = 2(A_1 - A_0)\eta \tag{A.5}$$

The above gradient for sphere boundary ($\eta = 1$) equals:

$$\left(\frac{\partial A}{\partial \eta}\right)_{\eta=1} = 2(A_1 - A_0) = 2A_1 - 5\overline{A} + 3A_1 = 5(A_1 - \overline{A}) \tag{A.6}$$

where the relation (A.4) is included.

Appendix B: Determination of A_1 in the proposed model

As a result of transformations of Eq. (46) an algebraic equation has been obtained:

$$aA_1^6 + bA_1^5 + cA_1^4 + dA_1^3 + eA_1^2 + fA_1 + g = 0 \tag{B.1}$$

where:

$$a = -3N_{rc}^* \tag{B.2}$$

$$b = 12N_{rc}^* A_i \tag{B.3}$$

$$c = \frac{18}{\pi\overline{A}} - 18N_{rc}^* A_i^2 \tag{B.4}$$

$$d = \pi^2 - (2c_1 + 3c_2 + 4c_3) + 12N_{rc}^* A_i^3 \tag{B.5}$$

$$e = \left(c_1 + 3c_2 + 6c_3 - \pi^2\right)\overline{A} - 6N_{rc}^* A_i\left(2A_i^2 - 3A_i + 2\right) + 3N_{rc}^* \tag{B.6}$$

$$f = -\frac{18}{\pi}\overline{A}^2 \tag{B.7}$$

$$g = c_3\overline{A}^3 \tag{B.8}$$

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